# A Higher-Order Topological Theory of Knowledge Recombination: When Do Breakthrough Innovations Emerge

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## Abstract

Breakthrough innovation arises from the recombination of existing knowledge, yet conventional dyadic network frameworks capture only pairwise links and therefore overlook the higher-order, many-to-many interactions through which novel combinations actually form. To address this gap, this study develops the Higher-Order Topological Theory of Knowledge Recombination (HOTTKR) to explain the emergence of breakthrough innovation beyond conventional binary network frameworks. Using USPTO patent data (1980–2021), we construct knowledge hypergraphs from CPC co-classifications and extract higher-order structural features through persistent homology, the simpliciality index, and the Hodge Laplacian, which jointly characterize the topological holes, connectivity, and flow structure of the evolving knowledge space. We propose the Topological Holes as Opportunity Windows (THOW) hypothesis and examine how topological holes, combinatorial maturity, and knowledge-flow efficiency jointly shape breakthrough innovation. The results show that proximity to topological holes significantly increases the likelihood of breakthrough innovation, Simpliciality exhibits an inverted U-shaped relationship with a tipping point at 0.522, beyond which excessive structural closure constrains novelty, and higher-order topological indicators provide predictive power beyond conventional dyadic network measures. In addition, the Hodge spectral gap leads breakthrough innovation by one to two periods, suggesting that shifts in higher-order topology serve as an early signal of impending breakthroughs. These findings advance a higher-order topological paradigm of innovation and introduce combinatorial readiness as a new theoretical construct, offering both a methodological toolkit for measuring the latent structure of knowledge and practical guidance for firms and policymakers seeking to anticipate and cultivate breakthrough innovation.

## Keywords

Knowledge Recombination, Breakthrough Innovation, Algebraic Topology, Hypergraph, Persistent Homology, Betti Numbers, Simpliciality, Hodge Laplacian